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The Use of the Correlation Method in Evaluating the Accuracy Characteristics of Precision Instruments

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Abstract—Evaluations of the accuracy characteristics of precision instruments obtained by the correlation method and the three-cornered hat method are compared. It is shown that the resulting evaluations are the same, and the correlation method is easier to implement and provides a more accurate assessment. A technique is proposed for evaluating the impact of the correlation of meter errors in the implementation of the correlation method. The results of natural and computational experiments (in relation to the measurement of the pulse sequence period) have shown that there is almost no correlation of the meter errors.

Keywords: precision measurements, evaluation of measurement errors, correlation of errors

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1. INTRODUCTION

The accuracy the characteristics of the instruments can be evaluated in tests that involve multiple measurements of the test/tested signal and the statistical processing of the measurement results [1, 2] in order to evaluate the instability source (generator) and/or the meter error. In general, the measured value T, the meter error a, and the measurement result A are random variables and for the ith dimension they are related by

$$A_i = T_i + a_i. \tag{1}$$

If the condition of the independence of the meter's error and the measured value holds true, the variance $D[A]$ of the results of multiple measurements is equal to the sum of the variance $D[T]$ of the measured value (the variance characterizes the instability of the generator of the measured value) and the variance $D[a]$ of the random component of the meter's error

$$D[A] = D[T] + D[a]. \tag{2}$$

In order to evaluate the meter's random error $D[a]$, the test signal generator is chosen so that the relation between the generator's instability and the random error of the meter satisfies the condition $D[T] \ll D[a]$ and then $D[a] \approx D[A]$. Conversely, in order to evaluate the instability of the generator $D[T]$, the meter satisfying $D[a] \ll D[T]$ is selected; then, the variance of the measurement results makes it possible to evaluate the instability of the test signal generator $D[T] \approx D[A]$.

A much more difficult task is the testing of precision equipment where the generator's instability and the meter's error are comparable and it is difficult to select equipment that satisfies the above relations. Because of this, the direct evaluation of the current noise characteristics of the generator and/or the meter's random error on the variance of the results of multiple measurements is impossible, and it is necessary to resort to indirect methods. One of these methods is the correlation method developed for measuring the jitter (short-term instability) of the period [3] and the time interval [4] of precision pulse generators. In order to implement the method, it is necessary that the period (time interval) is measured simultaneously by two time analyzers (**A** and **B**) as shown in the block diagram in Fig. 1.

The simultaneousness of the measurement is provided by the common start measurement signal fed to the triggering inputs of the meters, and the node splitter branches the test generator signal to the inputs of the meters. The measurement results are entered into a computer for analysis and display.

In [3] and [4], it is shown that the jitter of the period or time interval of the measured value generator can be evaluated according to the covariance of the results of simultaneous measurements:

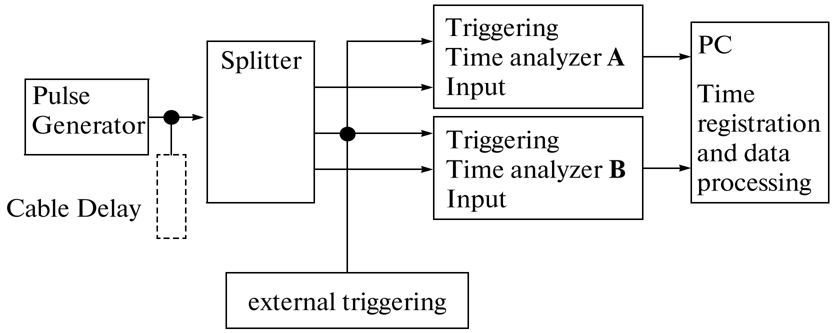

Fig. 1. A block diagram of the measuring system for the implementation of the correlation method. The dashed line shows the delayed pulse generator at the open segment of the coaxial cable.

$$D_2[T] = \mathrm{cov}(A, B) - \mathrm{cov}(a, b), \tag{3}$$

where $D_2[T]$ is the variance characterizing the instability of the generator of the measured value and determined by the correlation method, and cov(a, b) is the covariance of the errors of two meters in the measurement of a period (time interval). It should be noted that it is convenient to characterize the jitter of the period or time interval not by the variance but by the root mean square (RMS) of the jitter ($\sigma_T = \sqrt{D[T]}$), which has the same dimensions as the characterized quantity.

If the errors of the meters are uncorrelated, i.e., cov(a, b) = 0, then (3) is simplified to

$$D_2[T] = \mathrm{cov}(A, B). \tag{4}$$

Using this, it is not difficult to identify from (2) the variances of the random components of the meter errors:

$$\begin{aligned} D_2[a] &= D[A] - \mathrm{cov}(A, B), \\ D_2[b] &= D[B] - \mathrm{cov}(A, B). \end{aligned} \tag{5}$$

The implementation of any method of testing the reliability of evaluations particularly in relation to the testing of precision measuring equipment is currently important. This paper deals with the problem of the reliability of evaluations obtained by the correlation method.

2. COMPARISON OF TWO METHODS OF STUDYING PRECISION EQUIPMENT

The easiest way to ensure the reliability of evaluations obtained in tests is when it is possible to obtain the same evaluations by another method. In this case, the famous three-cornered hat method can be used [5]. This method is widely used in the study of the noise characteristics of signals in different fields of science and technology. In particular, it can be adequately used to evaluate the errors of three meters simultaneously measuring the same value. Unlike the correlation method, which requires two meter, this method requires three meters **A**, **B**, and **C**, which significantly increases the hardware and computational costs.

The method makes it possible to calculate (assuming the uncorrelatedness of the meter errors) the variance of the random error of each meter using the variance of the measurement result differences of the same input variable:

$$\begin{aligned} D_3[a] &= (D[A-B] + D[A-C] - D[B-C])/2, \\ D_3[b] &= (D[A-B] + D[B-C] - D[A-C])/2, \\ D_3[c] &= (D[A-C] + D[B-C] - D[A-B])/2, \end{aligned} \tag{6}$$

where $D_3[a]$, $D_3[b]$, $D_3[c]$ are the random error variances of the meters **A**, **B**, and **C**, respectively, defined by the three-cornered hat method; and $D[A-B]$, $D[A-C]$, $D[B-C]$ are the variances of the differences of the measurements results of the corresponding pairs of meters. In the case of the stationarity of the generator noises (in tests, this condition is usually met), according to (2), it is possible to evaluate the variance $D[T]$, which characterizes the instability of the generator input

Table 1. The comparison of natural experiment data obtained using the covariance method and the three-cornered hat method

Value	$\bar{D}[T]$ ps^2	σ_T ps	$\bar{D}[a]$ ps^2	σ_a ps	$\bar{D}[b]$ ps^2	σ_b ps
Set of meters 1						
Correlation method	6.79	2.61	9.52	3.09	11.20	3.35
Three-cornered hat method	6.73	2.59	9.73	3.12	11.32	3.36
Set of meters 2						
Correlation method	7.13	2.67	8.33	2.89	9.21	3.03
Three-cornered hat method	7.17	2.68	8.36	2.89	9.29	3.05

In the table notation, $\sigma_T = \sqrt{D[T]}$, $\sigma_a = \sqrt{D[a]}$, and $\sigma_b = \sqrt{D[b]}$.

$$\begin{aligned} D_{3A}[T] &= D[A] - D[a], \\ D_{3B}[T] &= D[B] - D[b], \\ D_{3C}[T] &= D[C] - D[c]. \end{aligned} \tag{7}$$

All three values should be approximately equal ($D_{3A}[T] \approx D_{3B}[T] \approx D_{3C}[T] = D_3[T]$) within the accuracy of the calculation of the variance for a finite number of tests.

The comparison of the methods was conducted both in the natural experiment and by means of computer simulation. In the natural experiment, the stability of the period of the pulse sequence of the generator AFG3251 made by Tektronix was investigated in the low jitter mode (when the sequence period is multiple to the period of the reference frequency of the generator). Precision event timer A033-ET [6] devices for measuring the event occurrence time (timing) were used as the meters. An event refers to the appearance of the pulse front; therefore, an event timer is a universal means of the temporal analysis of pulse sequences. Three timers were used to simultaneously measure the generator periods. According to (4) and (5), the variance of the generator period and the variance of the random errors of the meters for the correlation method were evaluated, while according to (7) and (6) the same evaluations for the three-cornered hat method were carried out. The evaluation was calculated using a series of 16000 measurements. The averaging of the evaluations and the determining the spread was carried out for approximately 500 series of measurements. Two sets of meters were used for the completeness checks.

Table 1 shows the mean variance $\bar{D}[T]$ and the RMS σ_T obtained in the natural experiment for the test signal generator period, the mean $\bar{D}[a]$ and the RMS σ_a of the random error of the timer **A**, and the mean $\bar{D}[b]$ and the RMS σ_b of the random error of the timer **B**.

The data of the natural experiment show that both methods make it possible to separate the components of the variance of the measurement results. At the same time, the RMS of the jitter of the AFG3251 generator's period is about 2.6 ps. The RMS of the random error of the timer **A** is 3.1 ps (2.9 ps for the second set of meters). The RMS of the random error of the timer **B** is 3.35 ps (3.04 ps for the second set of meters). The difference of the evaluations obtained by the two methods does not exceed 1%.

Evaluations obtained by the correlation and three-cornered hat methods were also compared using computer simulation. The reference conditions for the computing experiment were chosen roughly corresponding to the conditions of the natural experiment. In the computer simulation of the problem of measuring points and intervals of time, the model of Palm random flow was used with independent and identically distributed intervals between the successive events. In the simulation, the normal law of the distribution was selected for all the random variables (the fluctuation of the event occurrence times, the time intervals between the events, the time delay, the meter errors, and the instability of the generator). As in the natural experiment, the evaluation was calculated for a series of 16000 measurements, and, in order to obtain sustainable results, about 500 series of observations were accumulated, which were used to determine the mean values of the variances $\bar{D}[T]$, $\bar{D}[a]$, and $\bar{D}[b]$ and the standard deviations (STD) $\sigma_{D[t]}$, $\sigma_{D[a]}$, and $\sigma_{D[b]}$ of these evaluations. The results of the computer simulation (Table 2) show that the obtained evaluations of the instability of the generator and random errors of meters for both methods are identical.

Table 2. The comparison of the data of the computer simulations obtained using the covariance method and the three-cornered hat method

Value	$\bar{D}[T]$ ps^2	$\sigma_{D[T]}$ ps^2	$\bar{D}[a]$ ps^2	$\sigma_{D[a]}$ ps^2	$\bar{D}[b]$ ps^2	$\sigma_{D[b]}$ ps^2
Experiment 1						
Correlation method	6.69	0.14	9.70	0.17	11.30	0.18
Three-cornered hat method	6.69	0.20	9.70	0.17	11.30	0.18
Experiment 2						
Correlation method	7.09	0.14	8.30	0.15	9.20	0.15
Three-cornered hat method	7.09	0.19	8.30	0.15	9.20	0.16

In Table 2, the source data for experiments 1 and 2 correspond to sets of meters 1 and 2 of the natural experiment.

The computer simulation makes it possible to compare the accuracy of the evaluations given by each of the methods when varying the initial values of the variances of the source and meter errors. To do this, we investigated the dependence of the resulting evaluations of the standard deviations $\sigma_{D[T]}$ and $\sigma_{D[a]}$ on the preset values $D[T]$ and $D[a]$ (Fig. 2).

For the variances $D[T]$, the correlation method yields significantly (approximately by 1.4 times) more accurate results (Fig. 2, top panel). This result can be confirmed analytically if (4) is converted to the form

$$D_2[T] = \mathrm{cov}(A, B) = \frac{1}{2}\big(D[A] + D[B] - D[A - B]\big) = \frac{1}{2}\big(D[A] + D[B] - D[a] - D[b]\big) \quad (8)$$

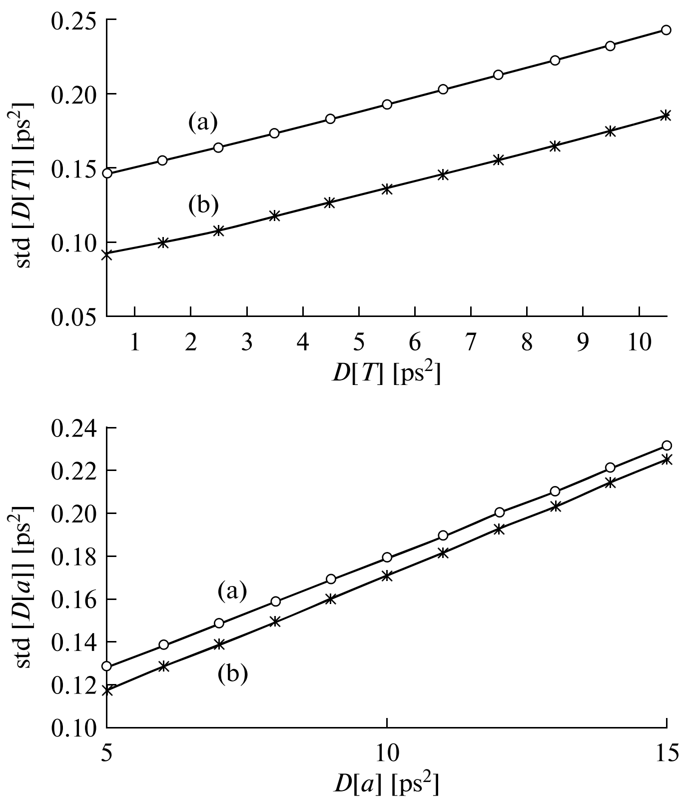

Fig. 2. The results of computer simulations on the dependence of the evaluations of the standard deviations of $D[T]$ (top panel) and $D[a]$ (lower panel) on the values of the variances. Lines (a) refer to the three-cornered hat method. Lines (b) refer to the correlation method.

Table 3. Comparison of the RMS errors of the evaluations obtained using the covariance method and the three-cornered hat method

Value	$\sigma_{D[T]}$ ps^2	$\sigma_{D[a]}$ ps^2	$\sigma_{D[b]}$ ps^2
Set of meters 1			
Covariance method	0.15 (0.14)	0.16 (0.17)	0.17 (0.18)
Three-cornered hat method	0.21 (0.20)	0.20 (0.17)	0.20 (0.18)
Set of meters 2			
Covariance method	0.16 (0.14)	0.16 (0.15)	0.17 (0.15)
Three-cornered hat method	0.21 (0.19)	0.19 (0.15)	0.20 (0.16)

In parentheses, the results of the computer simulation are given.

and substituted in (7)

$$D_2[T] = \frac{1}{2}\left(D_{3A}[T] + D_{3B}[T]\right). \quad (9)$$

Thus, the evaluation obtained by the correlation method is the mean of two evaluations obtained by the three-cornered hat method. Thus, the RMS error of the spread of the variance evaluations decreases by $\sqrt{2}$ times. The values $\sigma_{D[a]}$ for both methods are about the same (Fig. 2, bottom panel). These conclusions are supported by the results of the natural experiment as shown in Table 3.

3. THE METHODOLOGY FOR EVALUATING THE EFFECT OF THE CORRELATION OF METER ERRORS

From (4), it follows that the method makes it possible to measure the instabilities of the generator up to cov(a, b), which characterizes the correlation of the random errors of the two meters. Typically, in metrology, the correlation of individual components of the error of a meter is considered, but, in this case, a little-investigated area is examined, i.e., the correlation of random errors of the two meters. It is hard to imagine what physical processes can cause any dependence of the errors of two separate devices connected only by their ground and power networks. Therefore, it is possible to assume the uncorrelatedness of the meter errors. However, we cannot rule out the existence of some very fine mechanisms of the correlation of the meter errors, especially when it comes to measuring with the utmost precision.

In order to identify the possible correlations of the random errors of the two meters, we will proceed from the fact that, in the simultaneous measuring of the common input signal, the variance of the difference of the results of these measurements depends on the meter error's covariance

$$D[A-B] = D[a-b] = D[a] + D[b] - 2\,\mathrm{cov}(a,b). \quad (10)$$

It follows that the covariance of the random errors of the two meters cov(a, b) can be found if the evaluations of the variances $D[a]$ and the random errors $D[b]$ are known. It was possible to obtain such evaluations for the case of measuring the period using event timers where the time of the occurrence (timing) of both generator pulses and the pulses delayed for τ were determined (Fig. 3) [7].

As shown in [7], the covariance of the measurement results of the period and the interval τ provides evaluations of the variances of the timer errors:

$$D[a] = 2\,\mathrm{cov}(A,\tau_A),$$
$$D[b] = 2\,\mathrm{cov}(B,\tau_B). \quad (11)$$

By substituting these expressions in (10), we find that the covariance of the errors of the two timers can be evaluated as

$$\mathrm{cov}(a,b) = \mathrm{cov}(A,\tau_A) + \mathrm{cov}(B,\tau_B) - 0.5D[A-B]. \quad (12)$$

The robustness of this technique was tested in natural and computational experiments.

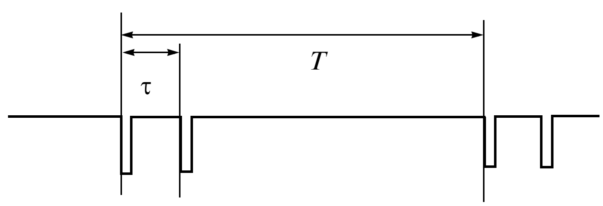

Fig. 3. The waveform diagram for the timer input signal.

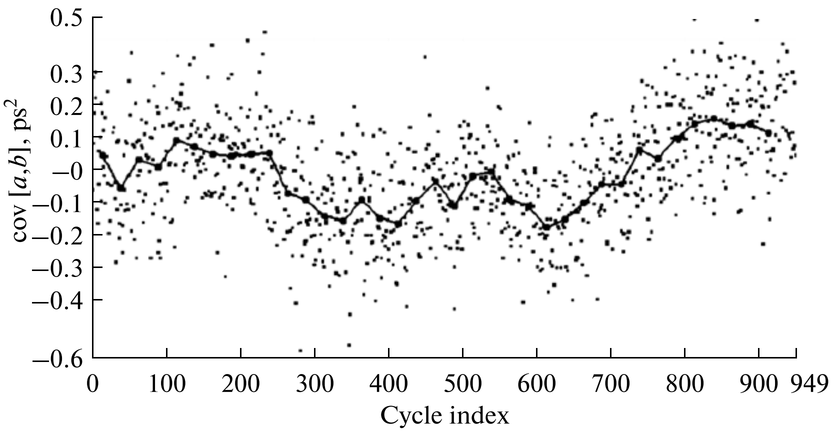

Fig. 4. Covariance of errors of the A033-ET timers. The points connected by the solid line show the averaged results of 25 cycles (16000 measurements in a cycle). The mean value across the whole sample is $\overline{\mathrm{cov}}\,[a,b] = -0.0109$ ps^2, $\sigma_{\mathrm{cov}[a,b]} = 0.16$ ps^2.

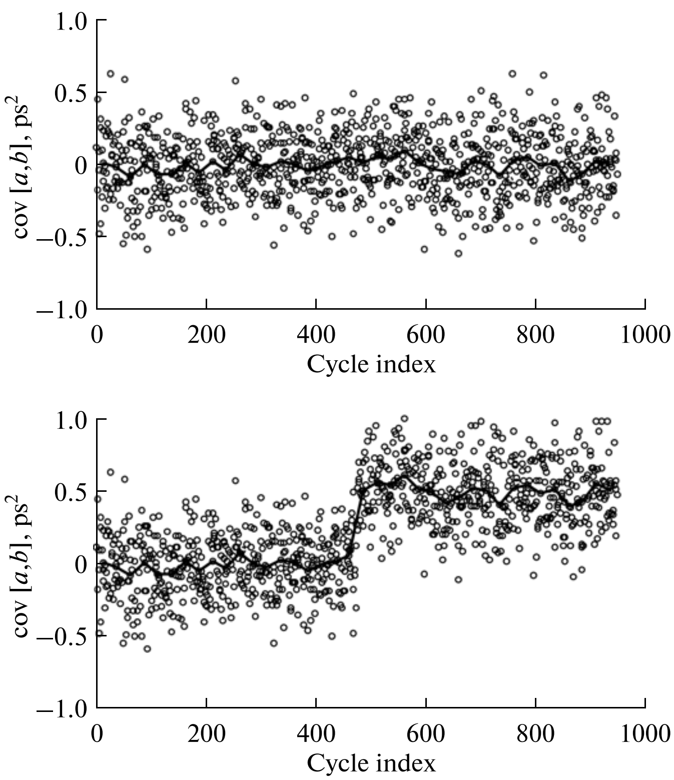

Fig. 5. The covariance of the meter errors in the computer simulation. The points connected by the solid line show the averaged results of 25 cycles (16 000 measurements in a cycle). The top graph shows the mean value of $\overline{\mathrm{cov}}[a,b] = 0.003$ ps^2, $\sigma_{\mathrm{cov}[a,b]} = 0.22$ ps^2. In the bottom graph, the correlation of errors of 0.5 ps^2 is introduced after 500 cycles.

In a modified version of the measuring system, the open segment of the broadband coaxial cable (shown in phantom in Fig. 1) was used for generating the delayed pulse [8]. The measuring result processing software calculates the periods of the test signal $\{A\}$, $\{B\}$ and the delay intervals $\{\tau_A\}$, $\{\tau_B\}$ for each of the event timers **A** and **B**, as well as the values common for the timers, i.e., the variance $D[A - B]$ of the difference of the measurements of one and the same period and the covariance $\mathrm{cov}(A, \tau_A)$, $\mathrm{cov}(B, \tau_B)$ of the measured periods and delay intervals.

For the natural experiment, A033-ET event timers (the RMS error of the period measurement was about 2.8 ps), a precision oscillator with an RMS of period jitter of 1.2 ps, and a broadband cable delay line ($\tau = 341$ ns) with instability of 1.65 ps were used. The results of the experiment are shown in Fig. 4.

The results of the natural and numerical experiments were compared on the basis of the same source data as in actual measurements. The mean interval between the events of the flow was set to 20.483997 s, and the mean time delay was set to 341 ns. For the Palm flow model, the variance of the random time interval between the events was set to $(1.2)^2$ ps^2. The delay jitter variance in the simulation of the event flows was set to $(1.65)^2$ ps^2. The timing error variance was set to $(2.8)^2$ ps^2.

The results are shown in Fig. 5, which shows how cov(a, b) changes from cycle to cycle. As can be seen from the graph, if the conditions of the uncorrelatedness of the meter errors is met, the value cov(a, b) determined in accordance with (9) is close to zero (Fig. 5, top panel). The computer simulation is convenient because it makes it possible to artificially introduce a certain degree of correlation of errors and identify its impact (as shown in Fig. 5, bottom panel).

According to the results of both the natural and computational experiments, cov (a, b) is close to zero (within the accuracy of the evaluations), which confirms the hypothesis of the uncorrelated random errors of the two meters.

4. CONCLUSIONS

In the study of precision equipment (such as meters and generators), the instability of the generator and the meter error are usually comparable, and the direct evaluation of these characteristics by the variance of the results of multiple measurements is impossible. When using indirect methods (in particular, the correlation method), the obtained evaluations should be validated.

The reliability of the correlation method was checked by the comparison with the evaluations given by the three-cornered hat method. Studies have shown that the evaluations are the same both in computing and natural experiments (within 1%). Moreover, the correlation method is not only easier to implement but it also gives more accurate evaluations.

The fulfillment of the meter error uncorrelatedness condition is a significant factor affecting the accuracy and reliability of the correlation method. The identification of the possible correlation of meter errors in general is difficult but, in the particular case, in relation to the pulse sequence measurement period, a procedure for such studies has been developed. The robustness of this technique is confirmed by the computer simulation, which showed the adequacy of the evaluations obtained at different degrees of correlation of the meter error. The results of the natural experiment, where A033-ET event timers were used as the meters, showed that the evaluation of the covariance of their errors gives values close to zero (within the accuracy of the evaluations). Thus, the hypothesis of the meter's random error's uncorrelatedness reflects the actual situation.

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REFERENCES

1. Ornatskii, P.P., *Teoreticheskie osnovy informatsionno-izmeritel'noi tekhniki*, (Theoretical Foundations of Information-Measuring Technique), Kiev: Vishcha Shkola, 1983.
2. *GOST* (State Standard) *R 54500.1/2011/ ISO/IEC: Guide 98-1:2009. Uncertainty of Measurenent. Part 1,* 2012.
3. Bespal'ko, V. and Bul', E., Estimation of test generator jitter with the help of time measurement devices, *Izmeritel'naya Tekhnika*, 2011, no. 6, pp. 59–62.
4. Bespal'ko, V., Boole, E., and Savarovskii, Ya., An instability estimation of precision time intervals, *Instrum. Exper. Techn.*, 2012, vol. 55, pp.377–382.
5. Riley, W.J., *Handbook of Frequency Stability Analysis*, Nat. Inst. Stand. Technol., NIST Special Publication, p. 1065.
6. Artyukh, V., Bespal'ko, E., Boole, V., and Vedin, V., . Advances of high-precision Riga event timers, *Proc. 16th Int. Workshop on Laser Ranging*, Poznan, Poland, 2009, vol. 2. pp. 398–403.
7. Bespal'ko, V.A. and Rybakov, A.S., Correlation method for estimation of event timing precision, *Autom. Control Comput. Sci.*, 2013, vol. 47, pp. 38–47.
8. Meleshko, E.A., *Bystrodeistvuyushchaya impul'snaya elektronika*, (Fast-Acting Pulsed Electronics), Moscow: FIZMATLIT, 2007.

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