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Correlation Method for Estimation of Event Timing Precision

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Abstract—A method for estimation of event timing precision is proposed and investigated. The method is based on a correlation analysis of the results of measuring the time coordinates of the original and delayed event flows. The timing error variance estimates are shown not to be directly dependent on the volatility of the time coordinates of the flows. Estimates are obtained for the variance of a random interval between events in the flow and the variance of the delay of the time-shifted flow. The estimates contain no components associated with measurement error. The capabilities of the method are illustrated by the results of computer simulations and actual measurements.

Keywords: event flow, correlation, event timing error, time interval
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1. INTRODUCTION

Physical experiments may sometimes (e.g., in the study of the interaction of radiation with matter [1]) encounter the problem of precise measurement and analysis of temporal and statistical characteristics of pulse sequences. Event flows, also called random point processes [2], can serve as a mathematical model of these sequences. In the context of pulse sequences, an event is associated with the time instant when the pulse's edge crosses a predetermined level. The time coordinates of an event flow are essentially the time instants of events and the time intervals between successive events. The most common way to describe the statistical characteristics of an event flow is to specify the multidimensional distribution densities of the time instants of events. Instead of time-instant distribution densities, one can use the multidimensional distribution densities of the time intervals between events [3]. It has been proved in the theory of stochastic flows that both ways of describing event flows are equivalent and one can always switch from the time-instant description to the time-interval description without losing any useful information [3].

In practice, when researchers analyze statistical properties of event flows, they are interested not in the distribution densities as such but in the numerical characteristics of the distributions, i.e., the expectation, variance, and covariance. The numerical characteristics of the distributions can be estimated by processing the information provided by measuring the time coordinates of an event flow, i.e., the time instants and intervals. The two event flow description methods suggest two methodological approaches to measuring the time coordinates of event flows:

—The continuous measurement of the time intervals between successive events with the subsequent calculation of the time instants (the interval measurement principle).

—The continuous direct measurement of the time instants of events (the event timing principle) with the subsequent calculation of the intervals between the events.

Each of the methodological approaches has led to specific hardware and software systems for conducting the measurements. For example, multistop time analysis systems [4] are based on the interval measurement principle, and precision event timers [5] are based on the event timing principle.

The actual flow of events is not directly available for observation and processing, but it is possible to record a statistically-associated observed flow of measurement results or estimates. It is assumed that there is some consistency between the time coordinates of the actual flow of events and the measured time coordinates of the observed flow. Measurements always deal with statistical estimates, and the magnitude of the measurement error determines the distortion of the temporal and statistical characteristics of the actual event flow. It has always been a difficult task to estimate the measurement error of high-precision equipment; this is also typical of event timers, provided that the current technology has allowed a reduction of the timing error to a few picoseconds [5].

Direct estimation of the timing error is complicated due to the lack of precise event-flow generators. The available precise time-interval generators [6] give an indirect estimate for the timing error through the measurement error for intervals between successive events (the standard error of interval measurements is divided by $\sqrt{2}$). This estimate for the timing error is admissible, but it has its limitations. First, there are always doubts about the actual relationship between the interval measurement error and the instability of the measured interval. Obviously, a reliable estimate is possible only if the variance of the interval measurement error is much greater that that of the measured interval. Second, a simple division by $\sqrt{2}$ is possible only if we know a priori that the timing errors of the start and stop events are uncorrelated. As a result, we have a very rough estimate for the timing error, and an expansion of the experimental data array does not improve the estimation accuracy.

In this paper, we propose a method for estimation of timing errors that does not impose strict requirements on the precision of the test generators. The method is based on the estimation of the correlation between two sequences of measured random variables. The first sequence is formed by the measured values of a random interval between successive events in the observed flow, and the second sequence consists of the measured delays of events in an auxiliary flow, which are detained in time with respect to the original flow events.

2. THEORETICAL PRINCIPLES OF THE METHOD

The idea of the proposed method for timing accuracy estimation is as follows. An event flow in the form of a pulse sequence is passed through a delay circuit. This generates an event flow that is delayed relative to the original flow by a (generally random) time shift τ. It will be shown below that the correlation between the measured time coordinates of the original and delayed event flows allows us to estimate the timing error variance.

The original (actual) event flow can be described by a sequence of time instants $\{t_k\}_{k=0}^{N}$ and a sequence of time intervals between events $\{T_k\}_{k=1}^{N}$, where

$$t_{k+1} = t_k + T_{k+1}, \quad k = \overline{0, N-1}; \tag{1}$$

$$T_k = t_k - t_{k-1}, \quad k = \overline{1, N}; \tag{2}$$

N is the total number of events; and the time instants are counted from the initial instant $t_0 = 0$. The elements of the set $\{T_k\}_{k=1}^{N}$ are realizations of the random variable T with an expectation of $\overline{T} = E[T]$ and variance of $D[T]$. The random variable T characterizes the interval between successive events in the actual flow, which is not distorted by timing errors. The random interval T can also be considered as a period of repetition (of a fluctuating length) of events in the flow; then, the variance $D[T]$ characterizes the period jitter. Note that here the period jitter is understood as fluctuations of the lengths of intervals between the time instants of events. The delayed event flow can be described by a sequence of time instants of events $\{\theta_k\}_{k=0}^{N}$, where

$$\theta_k = t_k + \tau_k, \quad k = \overline{0, N}, \tag{3}$$

and τ_k is the value of the delay for the kth event, which is a realization of the random variable τ with an expectation of $\overline{\tau} = E[\tau]$ and variance of $D[\tau]$. The random variables T and τ are assumed to be uncorrelated; i.e.,

$$\operatorname{cov}(T, \tau) = 0. \tag{4}$$

The average value of the delay should be such that the condition $t_k < t_k + \overline{\tau} < t_{k+1}, \quad \forall k = \overline{0, N-1}$ is satisfied (see Fig.1).

The time instants of the original and delayed event flows are measured using an event timer. The event timing results are sequences of measured data (estimates) for the time instants of the observed flow $\{\hat{t}_k\}_{k=0}^{N}$ and delayed flow $\{\hat{\theta}_k\}_{k=0}^{N}$. Here and below, the symbols denoting the measured values and estimates are

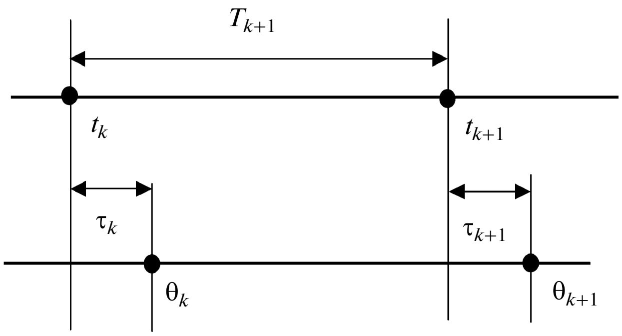

Fig. 1. Time coordinates of successive events of the original and delayed flows.

marked with a "circumflexus." The measured time instants (with timing errors) can be described by the following relations:

$$\hat{t}_k = t_k + \xi_k, \tag{5}$$

$$\hat{t}_{k+1} = t_{k+1} + \xi_{k+1}, \tag{6}$$

$$\hat{\theta}_k = \theta_k + \eta_k, \tag{7}$$

where ξ_k, ξ_{k+1}, and η_k are timing errors. Here, the timing errors of events of the original and delayed flows are denoted, for clarity, by different Greek letters, ξ (*xi*) and η (*eta*), but each of these errors is a realization of a random variable ε with an expectation of $\bar{\varepsilon} = E[\varepsilon] = 0$ and a variance of $D[\varepsilon] = \sigma_\varepsilon^2$. Here, we assume that the timing errors are not correlated; i.e., the following relationships for covariance are true:

$$\operatorname{cov}(\xi_i, \xi_j) = \operatorname{cov}(\eta_i, \eta_j) = \sigma_\varepsilon^2 \delta_{ij}, \quad \operatorname{cov}(\xi_i, \eta_j) \equiv 0, \tag{8}$$

where $\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$ is the Kronecker delta. The measured values for the time instants can be used to estimate the intervals between successive events and the delay of events in the delayed flow

$$\hat{T}_{k+1} = \hat{t}_{k+1} - \hat{t}_k, \quad k = \overline{0, N-1}, \tag{9}$$

$$\hat{\tau}_k = \hat{\theta}_k - \hat{t}_k, \quad k = \overline{0, N-1}. \tag{10}$$

Considering relations (2), (5), (6), and (7), the estimates for the time intervals between events and the delay of events in the delayed flow can be written as:

$$\hat{T}_{k+1} = (t_{k+1} + \xi_{k+1}) - (t_k + \xi_k) = T_{k+1} + \xi_{k+1} - \xi_k, \quad k = \overline{0, N-1}, \tag{11}$$

$$\hat{\tau}_k = (\theta_k + \eta_k) - (t_k + \xi_k) = \tau_k + \eta_k - \xi_k, \quad k = \overline{0, N-1}. \tag{12}$$

The elements of the set $\{\hat{T}_k\}_{k=1}^{N}$ are realizations of the random variable $\hat{T}$, which characterizes the period of the repetition of events in the observed flow (measured with random errors). The elements of the set $\{\hat{\tau}_k\}_{k=0}^{N-1}$ are realizations of the random variable $\hat{\tau}$, which has the meaning of a random delay measured with random errors. Considering relations (4) and (8), the variance of the random repetition period of the observed flow events can be written as

$$D[\hat{T}] = D[T] + 2\sigma_\varepsilon^2, \tag{13}$$

and the variance of the random delay measured with random errors can be written as

$$D[\hat{\tau}] = D[\tau] + 2\sigma_\varepsilon^2. \tag{14}$$

The differences between estimates (11) and (12), which can be written as

$$\hat{T}_{k+1} - \hat{\tau}_k = (T_{k+1} + \xi_{k+1} - \xi_k) - (\tau_k + \eta_k - \xi_k) = (T_{k+1} - \tau_k) + \xi_{k+1} - \eta_k, \quad k = \overline{0, N-1}, \tag{15}$$

can be considered as a realization of a difference random variable $\hat{T} - \hat{\tau}$, the variance of which can be described by two equivalent relations:

$$D[\hat{T} - \hat{\tau}] = D[\hat{T}] + D[\hat{\tau}] - 2\operatorname{cov}(\hat{T}, \hat{\tau}) = (D[T] + 2\sigma_\varepsilon^2) + (D[\tau] + 2\sigma_\varepsilon^2) - 2\operatorname{cov}(\hat{T}, \hat{\tau}), \tag{16}$$

$$D[\hat{T} - \hat{\tau}] = D[T - \tau] + 2\sigma_\varepsilon^2 = D[T] + D[\tau] - 2\operatorname{cov}(T, \tau) + 2\sigma_\varepsilon^2. \tag{17}$$

Subtracting (17) from (16) and considering (4), we get the equality

$$2\sigma_\varepsilon^2 - 2\operatorname{cov}(\hat{T}, \hat{\tau}) = 0, \tag{18}$$

from which, considering (13) and (14), we derive relations for the variance of the measurement error for the time instants of events (the timing error variance)

$$D[\varepsilon] = \sigma_\varepsilon^2 = \operatorname{cov}(\hat{T}, \hat{\tau}), \tag{19}$$

for the variance of the repetition period of the actual flow events

$$D[T] = D[\hat{T}] - 2\operatorname{cov}(\hat{T}, \hat{\tau}), \tag{20}$$

and for the variance of the delay

$$D[\tau] = D[\hat{\tau}] - 2\operatorname{cov}(\hat{T}, \hat{\tau}). \tag{21}$$

Both the measured intervals between events in the observed flow and the measured values for the delay contain the same timing error, which explains their correlation: $\operatorname{cov}\left(\hat{T}, \hat{\tau}\right) \neq 0$.

Thus, the correlation between the measured values of the intervals between the observed flow events and the measured delays of the delayed flow events allows us to directly estimate the variance of the actual flow's period jitter and the variance of the fluctuations in the delay produced by the delay circuit.

To calculate the estimates for the covariance $\operatorname{cov}(\hat{T}, \hat{\tau})$ and the variances (19), (20), and (21), we use the event timing experimental data samples collected in the arrays of the measured time instants of the original $\{\hat{t}_k\}_{k=0}^{N}$ and delayed $\{\hat{\theta}_k\}_{k=0}^{N}$ flows. Using (9) and (10), we calculate the elements of the arrays $\{\hat{T}_k\}_{k=1}^{N}$ and $\{\hat{\tau}_k\}_{k=0}^{N-1}$, which are used further in the formulae for unbiased estimates for the covariance and variances. The covariance $\operatorname{cov}(\hat{T}, \hat{\tau})$ of the random variables $\hat{T}$ and $\hat{\tau}$ is estimated by the formula

$$\hat{R}(\hat{T}, \hat{\tau}) = \frac{\sum_{k=0}^{N-1} (\hat{T}_{k+1} - \bar{\hat{T}})(\hat{\tau}_k - \bar{\hat{\tau}})}{N-1}, \tag{22}$$

the variance of the random variable $\hat{T}$ is estimated by the formula

$$\hat{D}[\hat{T}] = \frac{\sum_{k=1}^{N} (\hat{T}_k - \bar{\hat{T}})^2}{N-1}, \tag{23}$$

and the variance of the random variable $\hat{\tau}$ is estimated by the formula

$$\hat{D}[\hat{\tau}] = \frac{\sum_{k=0}^{N-1} (\hat{\tau}_k - \bar{\hat{\tau}})^2}{N-1}, \tag{24}$$

where $\bar{\hat{T}}$ is the sample mean for the array $\{\hat{T}_k\}_{k=1}^{N}$ of the measured intervals between the events in the observed flow

$$\bar{\hat{T}} = \frac{\sum_{k=1}^{N} \hat{T}_k}{N}, \tag{25}$$

and $\bar{\hat{\tau}}$ is the sample mean for the array $\{\hat{\tau}_k\}_{k=0}^{N-1}$ of the measured values of the delay of events in the delayed flow with respect to the original flow

$$\bar{\hat{\tau}} = \frac{\sum_{k=0}^{N-1} \hat{\tau}_k}{N}. \tag{26}$$

Variance estimates (19), (20), and (21) can be calculated using the following relations:

—The timing error variance estimate:

$$\hat{D}[\varepsilon] = \hat{R}(\hat{T}, \hat{\tau}). \tag{27}$$

—The variance estimate for the period jitter of the actual flow events:

$$\hat{D}[T] = \hat{D}[\hat{T}] - 2\hat{R}(\hat{T}, \hat{\tau}). \tag{28}$$

—The variance estimate for the delay jitter:

$$\hat{D}[\tau] = \hat{D}[\hat{\tau}] - 2\hat{R}(\hat{T}, \hat{\tau}). \tag{29}$$

3. COMPUTER SIMULATION RESULTS

The possibilities and limitations of the proposed method for timing precision estimation were investigated by the computer simulation of the problem of measuring the time coordinates of event flows. The effectiveness of the method was tested for two classes of random event flows: a quasi-periodic flow with independent jitter of time instants and a Palm flow with independent identically distributed intervals between successive events. The Palm flow is a flow with limited aftereffects and is also referred to in the literature as a flow with accumulation of time instant variance. The simulation used the normal distribution law for all the random variables (the time instant jitter, time intervals between events, time delay, and timing error). The average value of the interval between the events was set at 52.05 μs; the average value of the time delay was 435.68 ns. The total number of continuously recorded events varied from 1000 to 100000.

For the quasi-periodic flow model, the variance of the time instant jitter was set at 16.8 ps^2, which corresponds to a standard error of the jitter of 4.099 ps. The variance of the jitter period was equal to the double variance of the time instant jitter, i.e., 33.9 ps^2 (which corresponds to a mean square value of the jitter of 5.82 ps). For the Palm flow model, the variance of the random time interval between events was set equal to the variance of the jitter of the quasi-periodic flow (33.9 ps^2). The variance of the delay jitter in the event flow simulation was set at 21.4 ps^2 (which corresponds to the mean square value of the jitter of 4.63 ps). The timing error variance was set at 3.4 ps^2 (which corresponds to a standard error of 1.844 ps).

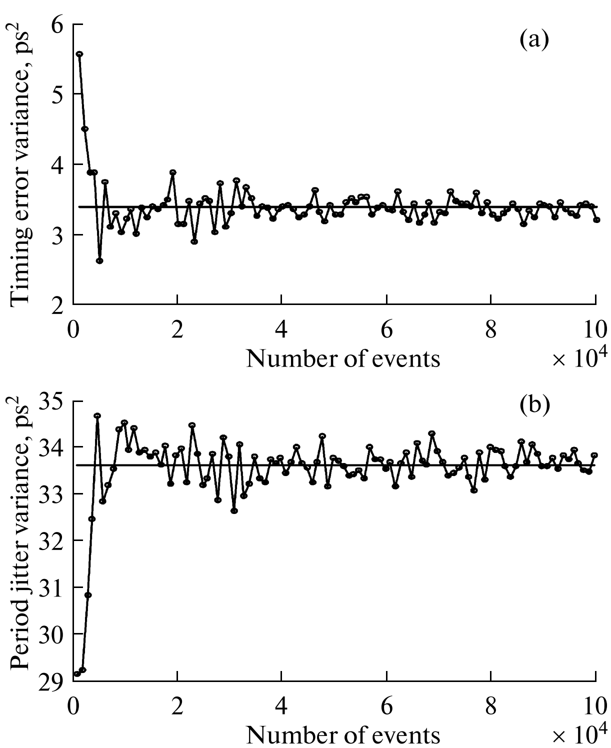

Fig. 2. Example of dependences of timing error variance estimates (*upper panel*) and period jitter variance estimates (*lower panel*) on the number of events (quasi-periodic event flow). The dashed horizontal lines show the given values of the variances (3.4 ps^2 and 33.9 ps^2).

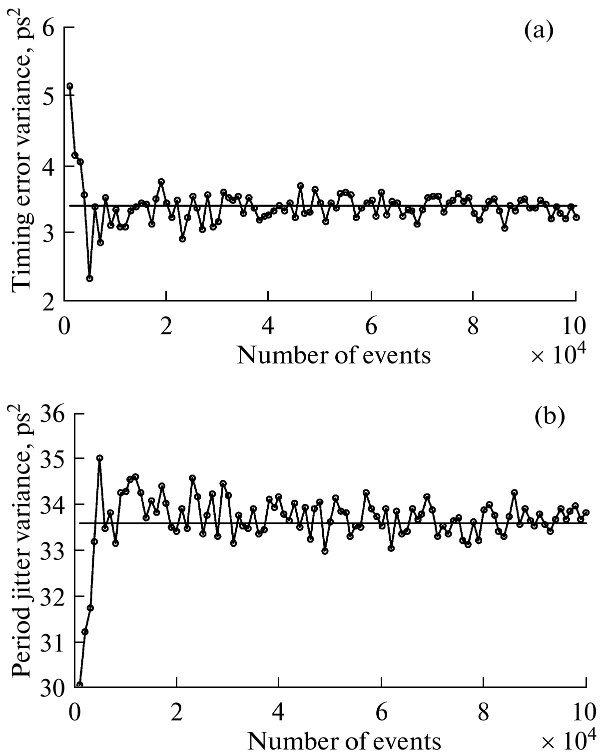

Fig. 3. Example of dependences of timing error variance estimates (*upper panel*) and period jitter variance estimates (*lower panel*) on the number of events (Palm flow). The dashed horizontal lines show the given values of the variances (3.4 ps^2 and 33.9 ps^2).

A numerical study was conducted of the dependence of the timing error variance estimates (27) and period jitter variance estimates (28) on the sample size (N), i.e., the number of recorded events. The results are plotted in Figs. 2 and 3. It should be noted that these empirical dependences of the variance estimates on N are sample realizations of random functions. Different realizations of the empirical dependences have different patterns, but all of them jitter around the true (set by the model) variance without going at $N > 20000$ beyond the interval of two to three sigmas.

The method for estimating variances (19), (20), and (21) was studied numerically by splitting the entire experimental data array $\{\hat{t}_k\}_{k=0}^{N}$ and $\{\hat{\theta}_k\}_{k=0}^{N}$ into groups and calculating the estimates for covariance (22) and variance (27), (28), and (29), which was followed by averaging the estimates across the groups. In fact, we were averaging the square deviations of the values from the mean, which was different in each group. This approach allows real-time (as soon as new events arrive) estimation of time error variances, period jitter variances, and delay jitter variances. The entire time-coordinate measurement process is divided into cycles with the experimental data array of one group being accumulated in one of the cycles.

We can judge about the effectiveness of the method of averaging the estimates calculated in each cycle by analyzing the dependence of the current averaged timing error variance estimate (given in the lower panel in Fig. 4) on the number of measurement cycles (2000 events are recorded in each cycle). Figure 5 shows the dependence of the current averaged timing error variance estimates, the period jitter variance estimates, and delay jitter variance estimates on the number of measurement cycles.

4. EXPERIMENTAL TEST RESULTS

An experimental study was conducted to test the timing error of the A033 event timer [7]. The overall procedure of the experiments is shown in Fig. 6. The input pulse sequence was produced by a pulse generator, and the time delay (of approximately 435 ns) was created by a precision monovibrator based on a D-type flip-flop with a cable delay in a feedback loop and a counter. Pulse generators of varying precision were used to generate the input sequence.

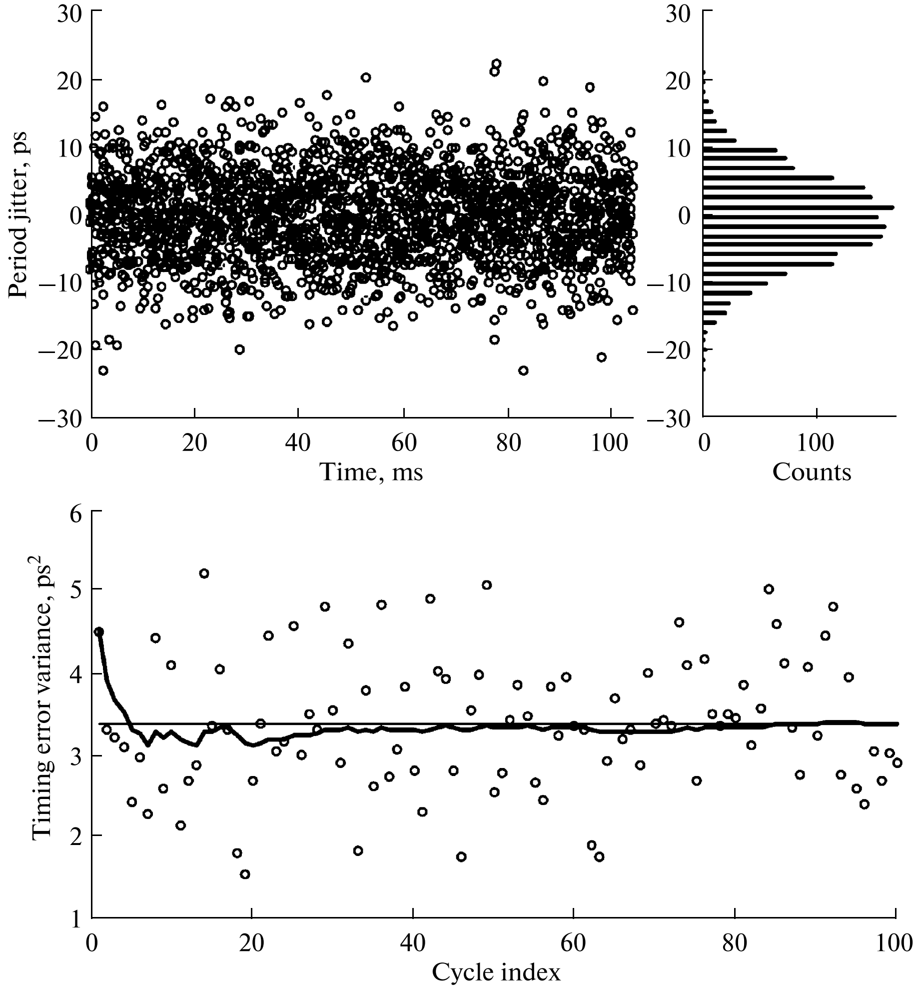

Fig. 4. Period jitter for events of the quasi-periodic flow in one cycle (*top left*), a distribution histogram for the period jitter (*top right*), and the dependence of the current averaged timing error variance estimate on the number of measurement cycles (*lower panel*). The circles in the lower panel show the variance estimates in each cycle. The dashed horizontal line shows the set values of the timing error variance (3.4 ps^2).

A software emulation of the circuit was created to run the experiments and process the results. The experimental results are reflected in the software emulation screenshots given below.

Figure 7 shows the results of an experiment whereby an AFG3252 Tektronix generator was used as a pulse generator.

The results are as follows: the generator period variance is $\hat{D}[\hat{T}] = 33.9$ ps^2, the delay variance is $\hat{D}[\hat{\tau}] = 21.4$ ps^2, and the 154-cycle average estimate for the timing error variance via the covariance is $\hat{D}[\varepsilon] = \hat{R}(\hat{T}, \hat{\tau}) = 3.35$ ps^2.

The range of variance estimates from cycle to cycle (the array size in each cycle is 16 000 events) is 2.49 ps^2 to 4.06 ps^2 (the max–min difference is 0.57 ps^2). The dashed line in the covariance graph shows the current averaged value of the covariance, and we can judge how well the averaging works over the repeating cycles. The graph for the current averaged covariance estimate is fundamentally consistent with the simulation results (Fig. 5, upper panel), and the input parameters for the simulation were selected so as to be similar to the actual experiment. If we had estimated the timing error variance using not the covariance (3.35 ps^2) but the measurements of the period (33.9/2 = 16.9 ps^2) or delay (21.4/2 = 10.7 ps^2), the estimate would have been seriously overestimated. The resulting timing error estimate ($\sqrt{3.35} = 1.83$ ps) can be used to estimate the interval measurement error for the À033 event timer, which is approximately 2.6 ps.

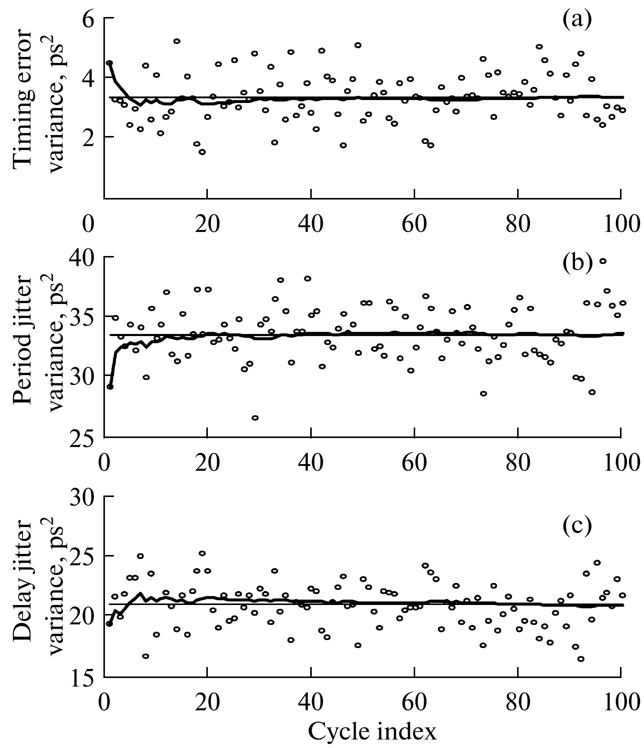

Fig. 5. Dependences of the current averaged estimates for the timing error variance (*upper panel*), the event repetition period jitter variance (*middle panel*), and the delay jitter variance (*lower panel*) on the number of measurement cycles. The circles show the variance estimates in each cycle. The dashed horizontal lines show the set values of the variances (3.4 ps^2, 33.9 ps^2, and 21.4 ps^2). It is a quasi-periodic event flow.

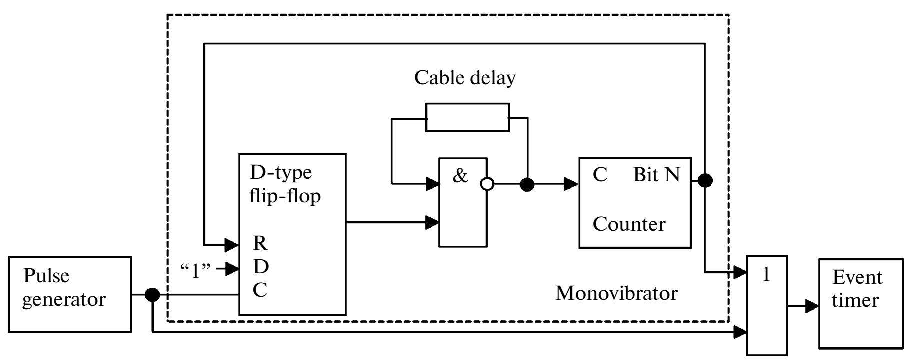

Fig. 6. Timing error experiment procedure.

Once we know the current error of the measuring instrument, we can not only monitor this error but also use these data in measuring some temporal characteristics, e.g., the period jitter of a precision pulse sequence. Figure 8 shows the results of an experiment on the study of the period jitter of the EXO-3 crystal oscillator.

According to (28), the period jitter of the oscillator can be estimated as follows:

$$\hat{D}[T] = \hat{D}[\hat{T}] - 2\hat{R}(\hat{T}, \hat{\tau}) = 10.4 - 2 * 3.25 = 3.9 \text{ ps}^2.$$

Thus, the period jitter of the EXO-3 oscillator is $\sqrt{3.9} = 1.97$ ps, which is a very good parameter for a standard crystal oscillator. If the oscillator's jitter were measured directly within the capabilities of the

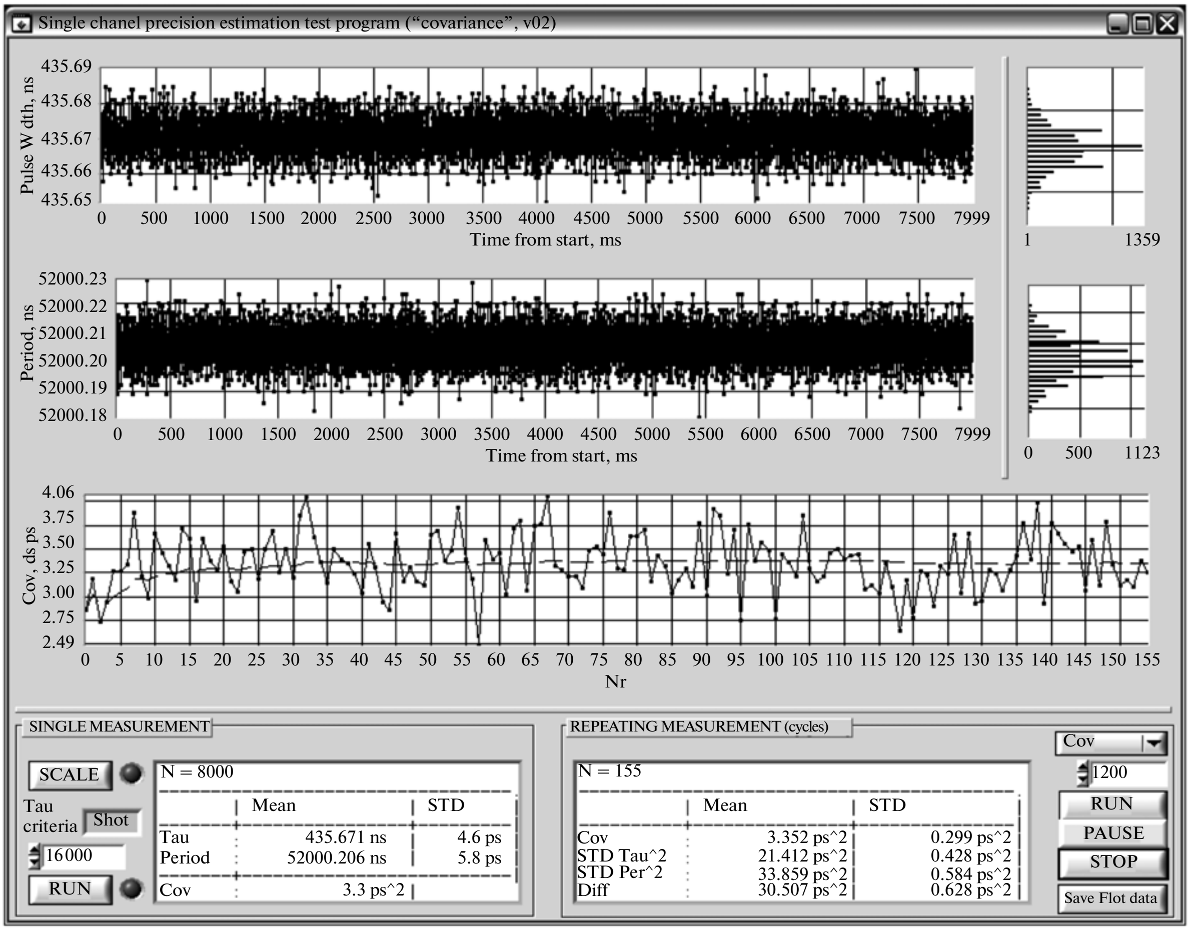

Fig. 7. Results of the timing error experiment with an AFG3252 generator.

A033 event timer, its estimate would be 3.22 ps; i.e., the measurements would yield a very rough estimate with an error greater than 60%.

5. CONCLUSIONS

A method was developed and investigated for estimating the timing error in the measurement of the time instants of events in their continuous flow. The method is based on the assessment of the correlation between two sequences of measured values of random variables. The first sequence is formed by the measured values of a random interval between successive events in the observed flow, and the second sequence consists of the measured delays of events in an auxiliary flow, which are detained in time with respect to those of the original flow.

The correlation between the measured intervals and the measured delays was found to estimate the timing error variance, the period jitter variance for the fluctuations of intervals between events in the flow, and the delay jitter variance.

The method was shown theoretically to ensure that the resulting estimates are relatively independent of the time volatility of the input pulse sequences. A distinctive feature of the method is the possibility to significantly increase the accuracy of the estimates by averaging the measurements. Numerical simulation techniques showed that several dozens of measuring cycles are sufficient to obtain reliable error estimates.

The capabilities of the method were confirmed by experimental studies, first, of the A033 event timer (the interval measurement error proved to be 2.6 ps) and, second, of the EXO-3 crystal oscillator (the period jitter proved to be approximately 2 ps).

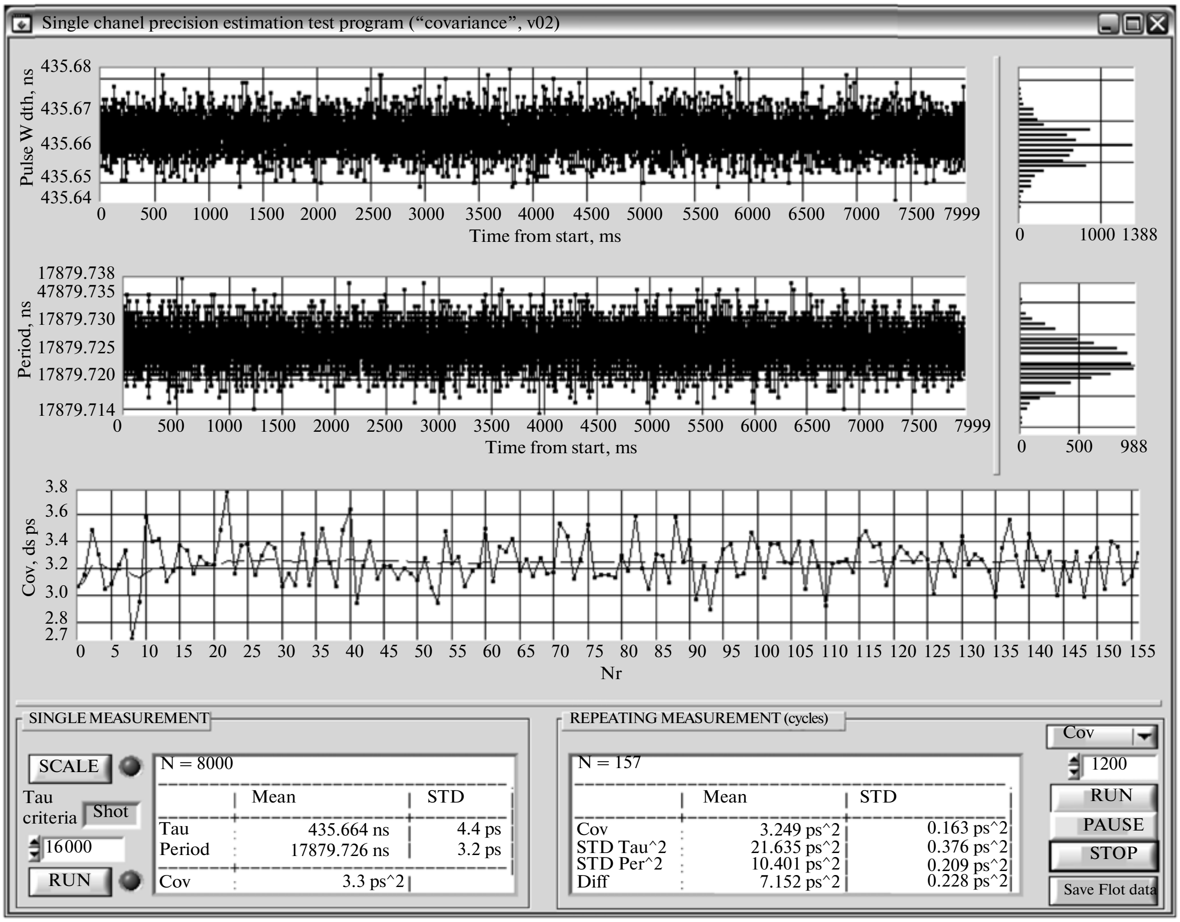

Fig. 8. Results of the study of the period jitter for the EXO-3 crystal oscillator with a frequency of 14.383 MHz and a denominator of 256.

ACKNOWLEDGMENTS

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