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An Instability Estimation of Precision Time Intervals

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Abstract—A method for estimating the instability of time intervals (TIs) is described, when they are measured simultaneously (in parallel) by two measuring units. The method is based on the calculation of the covariance of parallel measurement results and statistical averaging and is intended to investigate instabilities in picosecond and subpicosecond ranges. The relative error of the method is estimated by a computing experiment and depends on the amount of the measured data and error values of the used TI meters. An example of the setup, which allows one to evaluate the TI instability up to 0.4 ps with a relative error of no more than 10%, is given. The experimental data on the TI instability in the picosecond (for AFG3252 two-channel generator, it varies from 2.4 to 5.2 ps for intervals of the microsecond range) and subpicosecond (for the TI generator based on a quartz voltage-controlled oscillator from the FORDAHL company, it is 0.86 ps) ranges are obtained.

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INTRODUCTION

The generation of time intervals (TIs), formed by two pulse trains, refers to the traditional procedure of the physical experiment. The TI sources are the time interval generators, delay generators, time shift sources, etc. Of special importance are the precise TI generators, on which stringent stability requirements for generated intervals are imposed. The TI instability value characterizes a level of random changes of the intervals, and for the precise generators it varies from tens of picoseconds (for the already out-of-date И1-8 time shift source) to a few picoseconds (for the T5300U up-to-date generator from the VIGO System S.A. company [1]).

It is rather difficult to estimate the actual instability of this level, but in principle it is possible, if such expensive equipment as a fast digital oscilloscope (with a bandwidth of at least 6–8 GHz) and special software are available. Simpler approaches for solving this problem, which are based on using the time measuring equipment, are further considered.

ESTIMATION OF THE TI INSTABILITY BY USING THE TIME ANALYZING EQUIPMENT

It is comparatively simple to obtain the TI instability estimation, if there are possibilities of their multiple measurements, storage of these measurement results, and their further mathematical processing. There are rather many devices for time measurements (time-interval analyzers, event timers, time digitizers, timer/analyzers, etc.), which possess these capabilities.

Assume that, by repeatedly measuring the studied TI with duration T, n measured A values were stored. Each measured A_i value of the studied time interval can be represented as a sum of two random values (measured interval T_i and its measurement error a_i):

$$A_i = T_i + a_i. \quad (1)$$

On the assumption of independence of the measurement error and the measured value, dispersion $D[A]$ for an array of the n measured values is equal to the sum of the dispersion of the studied time interval $D[T]$ and dispersion of the measuring unit error $D[a]$:

$$D[A] = D[T] + D[a]. \quad (2)$$

The estimation of the TI instability, which is usually characterized by the STD value (standard deviation $\sigma = \sqrt{D[T]}$), does not present difficulties, if it is so much larger than the error of the meter, that the latter can be neglected. Then, $D[A] \approx D[T]$, and $\sigma = \sqrt{D[A]}$ acts as the TI instability estimation. Figure 1 shows an example of this situation.

The object under study was the time interval generated by the И1-8 time shift source, whose instability estimate was 46 ps, and the meter was the A033-ET event timer [2], having a measurement error of a single TI smaller than 5 ps.

The error of the instability estimation depends on the number n of measurement results. As it is known, when the distribution law is normal, the confidence interval dispersion D in the statistical test method is determined as [3]

$$I_\beta = (D - t_\beta \sigma_D; D + t_\beta \sigma_D), \quad (3)$$

where $t_\beta = 2.5$ with a confidence probability of 0.99, and $\sigma_D = \sqrt{\frac{2}{n-1}} D$.

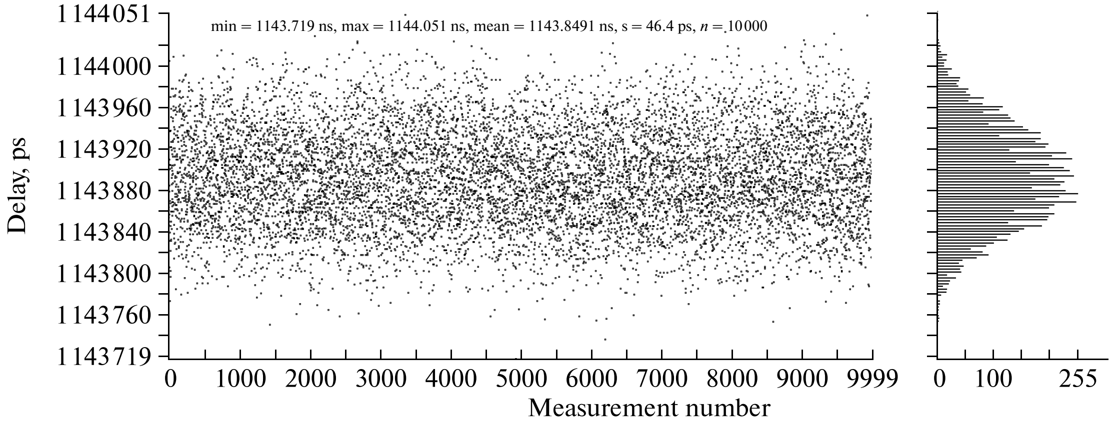

Fig. 1. Measured instability of TIs with durations of 1143 ns, generated by the И1-8 time shift source (measured TI train is on the left, and the measured data distribution histogram is on the right).

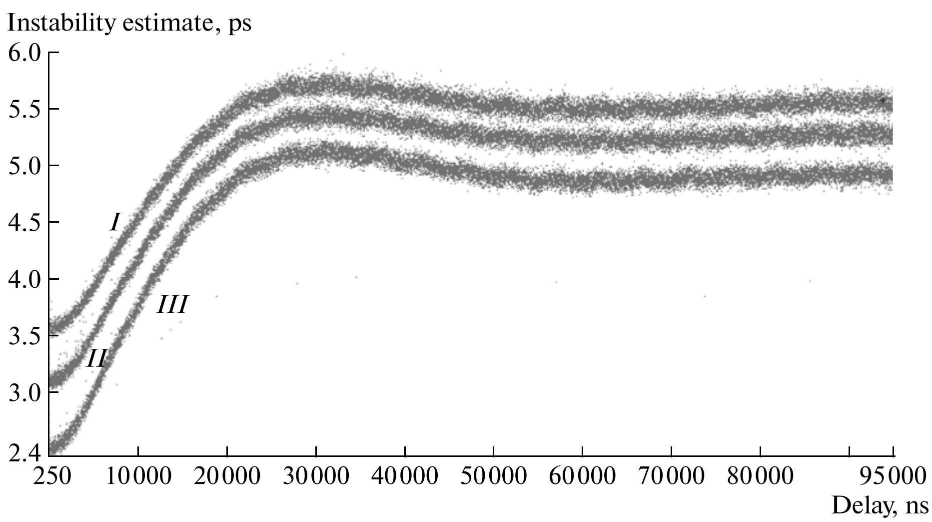

Fig. 2. Instability estimate of the delay between channels of the AFG3252 generator in a delay range of 250 ns–95 μs (the step is 5 ns, n = 3000): (*I*) from the dispersion of measurement results obtained by one meter; (*II*) from the dispersion of the half-sum of the simultaneous measurement results by two meters, and (*III*) from the covariance of the simultaneous measurement results by two meters. The TI repetition period is 100 μs.

The relative error of the dispersion estimation will be $t_\beta \sigma_D$, and it is possible to show that for the instability estimation of the interval σ, the relative error will be approximately two times smaller. When the amount of measured data is sufficient, it is not difficult to obtain a relative error of the TI instability estimation within several percents (thus, when $n = 3000$, the error of the instability estimation will not exceed ±3.2%, and, when $n = 10000$, it will be ±1.8%).

The problem of obtaining the TI instability estimates is substantially complicated, when the instability value is comparable with the error of the available meter. Figure 2 shows results of investigating the TI instability, the source of which is the AFG3252 two-channel arbitrarily-shaped signal generator (Tektronix Co.). The generator is intended to change the TI duration in a wide range by the controlled delay between the channels. As a meter, the A033-ET timer was used. The generator and meter were controlled from one computer. This allows one to synchronize the measured data storage cycle (n measured data were stored in one cycle) with changing to the next interchannel delay value.

Each point in the obtained dependence *I* in Fig. 2 is the estimate of the $\sigma = \sqrt{D[A]}$ value from 3000 measurement results of one TI, determined by the speci-

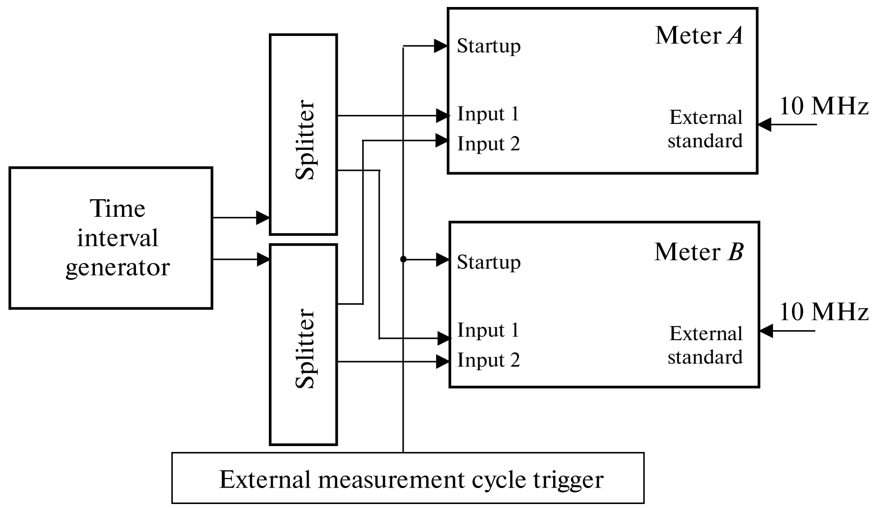

Fig. 3. Block diagram for parallel measurements by two meters.

fied delay between the channels of the AFG3252 generator. The first conclusion, which follows from the presented results, is that the instability of the interchannel delay of the AFG3252 generator is comparable with the error of the used meter, and, in the studied delay ranges, it varies from 3 to 6 ps. Additional investigations have shown that this instability level is preserved only, if the TI repetition period is divisible by 100 ns (this is the period of signals of the reference generator), but even taking into account this limitation, possibilities of using the AFG3252 generator are significantly wider than it is determined by its specification. The second conclusion is that there is a certain dependence of the delay instability on its delay value, but, reasoning from (2), it is difficult to say, whether it is related to special features of the source or meter.

As a result, due to a commensurability of the TI instability estimate and error value of the meter, the estimate proves to be rather coarse and more qualitative than quantitative. One should face the fact that the measurement error of a single time interval of 3–5 ps is not limiting for the up-to-date levels of the time analyzing equipment [2, 4].

TI INSTABILITY ESTIMATION FROM THE DISPERSION OF THE HALF-SUM DURING PARALLEL MEASUREMENTS

It is possible to actually increase the accuracy of the TI instability estimation, if the measurements are performed by several meters in parallel [5]. The block diagram for performing these measurements by two meters is shown in Fig. 3. The studied TIs are measured in parallel (simultaneously) by two meters *A* and *B* (e.g., the above-mentioned A033-ET event timers can be used as meters). The simultaneity is ensured by the external trigger of the measurement cycle of both meters. To match correctly the loads, the studied TIs arrive at inputs 1 and 2 of the meters through splitters (each splitter is a high-speed comparator with branched outputs intended for operation into a 50-Ω load). The computer controls operation of the meters, stores measurement results, and further processes them.

In each measurement cycle, each meter records n TI measurements, allowing one to calculate different statistic characteristics of these results. It is possible to obtain the TI instability estimation by calculating the dispersion of the half-sum $(A + B)/2$ of simultaneous measurement results, allowing one to average errors of the meters [5]. In the general case:

$$\begin{aligned} &D[(A+B)/2] \\ &= D[T] + (D[a] + D[b])/4 + \mathrm{cov}[a,b]/2. \end{aligned} \quad (4)$$

The error-decreasing effect during averaging depends on correlation of the errors of the meters (when the errors are completely independent, the effect is at maximum, and, when the errors are completely correlated, there is no reduction at all, and expression (4) changes to (2).

The experimental investigations confirm that the TI instability estimation from the dispersion of the half-sum of the measurement results by two meters (dependence *II* in Fig. 2) gives a gain in the accuracy, as compared with the use of one meter (dependence *I*). However, as to the dependence of the delay instability on its value, here it is difficult to solve, whether one should refer it to features of the source or meter.

ESTIMATION OF THE TI INSTABILITY VIA THE COVARIANCE OF THE SIMULTANEOUS MEASUREMENT RESULTS

A more accurate estimate of the picosecond and subpicosecond precision TI instability can be obtained, if the TI instability is evaluated via the covariance of the simultaneous measurement results. It was shown in [6] that the covariance of the periods of the tested pulse train, measured simultaneously by two meters allows one to estimate the period jitter. These results are also completely applied to the TI instability estimation, i.e., the covariance of the simultaneous measurement results of the interval by two meters is equal to the sum of the dispersion of the TI source $D[T]$ and covariance of the measurement errors a and b of meters A and B, respectively:

$$\mathrm{cov}[A,B] = D[T] + \mathrm{cov}[a,b], \qquad (5)$$

where cov[A, B] is calculated from the measurement results stored in the cycle.

The cov[a, b] value is unknown, but if we assume the mutual independence of the errors of the meters, then cov[a, b] = 0 and the covariance value gives the TI instability estimation:

$$\sigma^2 = D[T] = \mathrm{cov}[A,B]. \qquad (6)$$

Dependence *III* in Fig. 2 reflects the results of the instability estimation of the delay between the channels of the AFG3252 generator from the covariance of the simultaneous measurement results (A033-ET timers were used in the measuring setup, their TI measurement error dispersion $D[a] \approx D[b] \approx 6$ ps^2, and n = 3000 measurements were performed in the cycle).

As it was shown in [6], a larger or smaller measurement error leads only to the fact that a larger or smaller scatter of the obtained estimates occurs. Therefore, the use of the covariance of the simultaneous measurement results, as an estimate of the TI instability, when TI instability is in fact estimated, without a contribution of the error of the meters, allows one to obtain the substantially lower estimate of the TI instability (for different delay values of the AFG3252 generator, it varies from 2.4 to 5.2 ps), and, in this case, the dependence behavior remains approximately the same (but the influence of the meters is already excluded).

The investigations of the TI with a subpicosecond instability most completely demonstrate the capabilities of the covariance-using method. Figure 4 shows investigation results of the instability of the TI generated by the precision time interval generator based on the quartz VCO from the FORDAHL Co. (jitter of the period indicated in the technical specifications is <1 ps).

These results show that when a small TI instability is estimated by both one meter (dependence *I*) and two meters from the dispersion of the half-sum (dependence *II*), eventually the estimate of the meters than of the TI source is obtained. It is only possible to obtain the subpicosecond TI instability estimate (for the studied generator, it varies from 0.75 to 1.00 ps) via the covariance of the simultaneous measurement results (dependence *III*). It is difficult to obtain the instability estimates of this level even by using fast digital oscilloscopes, e.g., the DSA90804A oscilloscope (bandwidth is 8 GHz) from the Agilent Co. during time-interval jitter measurements have 2-ps intrinsic noises.

ANALYSIS OF THE FACTORS INFLUENCING THE ERROR OF THE TI INSTABILITY ESTIMATION VIA THE COVARIANCE OF THE SIMULTANEOUS MEASUREMENT RESULTS

The analytic expressions of the dependence of the error of the obtained estimates on the value of the studied magnitude, accuracy of the meters, and averaging volume are cumbersome and little informative [7]. At the same time, the measurement process is sufficiently simply simulated. Three arrays of random numbers are generated in one cycle with a normal distribution and variable root-mean-square deviations. One array, which is the model of the studied TI instability, is summed with each array, simulating errors of the meters, and the covariance of the simultaneous measurements is calculated for these two obtained sums.

Figure 5 shows the operation of the model, in which the actual measurement results of the TI instability, generated by the generator based on the quartz VCO from the FORDAHL Co. (Fig. 5a), and results of the computing experiment with the model (Fig. 5b) are compared, and, in this case, the parameters of the model were the same, as in the measuring setup, i.e., $D[a] \approx D[b] \approx 6$ ps^2, and n = 3000. The results are shown as the distribution function of the instability estimates with the assumption of the multiple cycle repetitions. This allows one to estimate the statistical scatter of the instability estimates and their error.

The TI instability estimation without averaging cycles was 0.86 ps, and the estimation error was 54 fs (from the model it is 76 fs); with the averaging over 10 cycles, the estimation error decreased to 23 fs (from the model, it is 24 fs); with the averaging over 100 cycles, the estimation error decreased 12 fs (from the model, 7 fs). The results show both the efficiency of averaging the measurement results, when the cycles are repeated, and the compliance of the model to the actual measuring setup.

These results also point to some limitations of the proposed method. The method is fundamentally statistical and calls for storing large data arrays; in this case, the higher the required accuracy of the estimate is, the larger the time burden is. Thus, the relative error of the instability estimation without cycle averaging is rather high (Fig. 5, m = 1), about ±20% (at a 3σ level). To increase the accuracy of the estimation up to ±5%, it is necessary to repeat measurement cycles at least 100 times (m = 100). Therefore, the method is most efficient, when the investigations of fast time-interval variations (jitters) are investigated in the nano- and microsecond ranges.

Figure 5b shows the simulation result only for one particular TI instability value (0.86 ps) of the studied source. By performing the computing experiment for different instability values, it is possible to simulate the dependence of the error of the TI instability estima-

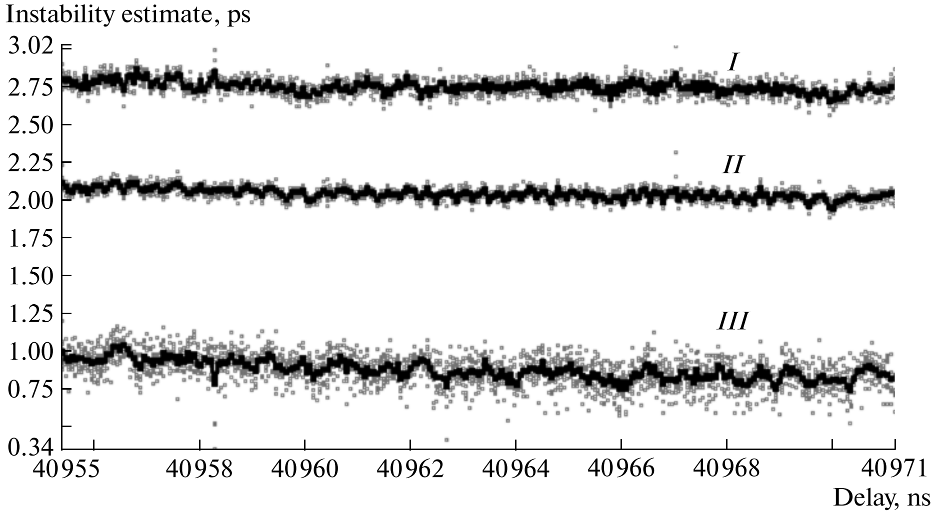

Fig. 4. Instability estimate of the time interval between channels of the generator based on the quartz VCO in an interval range from 40.955 to 40.971 μs (the step is ~62 ps, $n = 3000$) from: (*I*) from the dispersion of measurement results obtained by one meter; (*II*) from the dispersion of the half-sum of the simultaneous measurement results by two meters, and (*III*) from the covariance of the simultaneous measurement results by two meters. The interval repetition period is 82.92 μs. The bold line distinguishes averaging over 10 cycles.

tion on the value of its instability (Fig. 6; parameters for the computing experiment were the same, as in the measuring setup).

Figure 6 shows the bounds of the relative estimation error from 1 to 50%, allowing one to determine the minimal instability determined with the specified relative error. Thus, if the specified relative error is no worse than 10%, then for $m = 1$, the minimally estimated instability is 1.3 ps, for $m = 10$, the measured instability range expands to 0.6 ps, and for $m = 100$, it does to 0.4 ps. These figures are the model estimates of the capabilities of the measuring setup, and it is possible to actually check them only when the TI generators of appropriate precision are available.

At first sight, by increasing the number of averaged cycles (m), it is possible to increase with no limit the instability estimation accuracy. This is not the case, since, according to (5), the estimation error contains the cov[a, b] value as a systematic error component, and, in addition, this component is equal to zero only if the measurement errors of the timers are independent (expression (6) is based on this assumption).

The experiments have shown that one of the most important factors influencing the correlation of the errors of the meters is the use of a common external reference frequency source (in Fig. 3, this corresponds to the use of a 10-MHz common reference frequency signal). In this case, the unstable nature of the resulting interrelations of the meter errors is reflected as a significant increase in the scatter of instability estimates. This factor is easily eliminated, if different external reference frequency sources are used for the meters.

Another important factor could be the influence of the single frequency of the alternating-current power system via the power supply units. However, this influence is typical for analog power sources, which are not virtually used now. The up-to-date pulse power supply

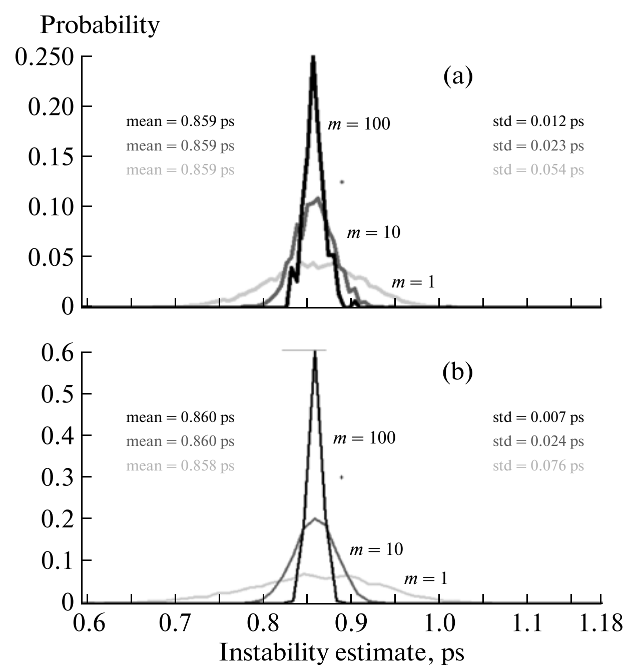

Fig. 5. Instability estimate of the time intervals generated by the generator based on the quartz VCO from the FORDAHL Co. (the TI value is 40.96 μs, the repetition period is 81.92 μs, and m is the number of averaged cycles): (a) actual measurements and (b) computing experiment.

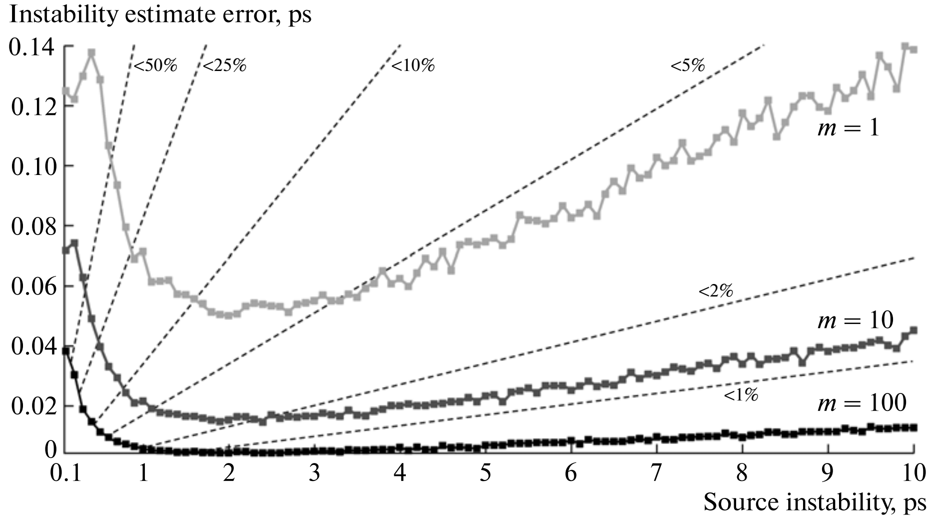

Fig. 6. Simulation results of the dependence of the error of the instability estimate on its value for $m = 1$, 10, and 100. The bounds of the relative estimate error are shown from 1 to 50%.

units have individual kilohertz operating frequencies, excluding the appearance of this error correlation mechanism.

In principle, difficulties with estimating the real cov[a, b] value are the certain limitation of the method, since it is possible to theoretically assume that there are some other thin mechanisms of correlations of the meter errors (e.g., related with their sensitivity to on-air noises). In any case, the revelation of both mechanisms of the error correlation of the meters and the significance of their influence is the subject of another study.

CONCLUSIONS

In our opinion, the method for estimating the TI instability from the covariance of the simultaneous measurement results allows one to study picosecond- and subpicosecond-level instabilities. The method is based on using the available equipment for time measurements and comparable in accuracy with using fast digital oscilloscopes. Possibilities of the particular measuring setup depend on parameters of the used meters (measurement error of the single TI, volume of the measurement results recorded in the cycle) and are easily determined from the model. When the cycles repeat, the efficiency of averaging the obtained estimates is high. The method is also applicable for studying other parameters of precision pulse trains, in particular, instability (jitter) of a period. Certain limitations imposed on the method are related to the need for accumulation of large amount of the measurement results.

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