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Statistical Approach to Linearity Evaluation of High-precision Event Timers

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Introduction

In recent years the technique of event timing becomes preferable for high-precision time interval measurement. Unlike conventional measurement of single-shot time intervals between Start and Stop pulses, the event timer measures time instants at which corresponding events occur. The events are associated with some specific points of input signal, e.g., with leading edges of input pulses. Then the time intervals between any pairs of events can be simply calculated.

The event timers have extended functional possibilities and provide much better precision (up to units of picoseconds) as compared with conventional time interval counters. These features are important for many applications but especially for Satellite Laser Ranging (SLR) where an extreme precision of wide-range measurement is vitally needed [1]. Currently the event timers are considered as the most suitable devices for advanced SLR applications.

Some years ago the high-precision event timers were not sufficiently high-speed, needing up to tens of microseconds for every single measurement. For this reason they are usually combined in multi-channel timing systems to increase the total operation speed. However, such timing systems are too expensive and time-consuming to build. The latter-day event timers become much faster [2]. They don't need more than 100 ns for every single measurement, allowing use of them in much simpler and cheaper single-channel configurations of timing systems. This kind of event timers will be meant in the following discussion concerning evaluation of their linearity.

The problem to be solved

Without regard to the time-base instability, the measurement error for time interval T_i between two adjacent events can be expressed as follows:

$$\Delta(T_i)=E(T_i)+\xi_i\,, \qquad (1)$$

where $E(T_i)$ – a systematic error that depends on the time interval value T, and ξ_i – some centered random error (so called measurement jitter). Unlike conventional time interval counters, the event timers actually do not produce any constant error (offset) if the measured events come at the single input. The function $E(T)$ defines the linearity of time interval measurement in a specified range of time interval variation. In this case the maximum value of this function, expressed in time units, is conventionally considered as a non-linearity measure for specific timing device.

In many cases the total measurement accuracy is dominated by the non-linearity. For example, the popular Time Interval Counter SR620 from "Stanford Research System" [3] has typical non-linearity of ±50 ps which is much more than its specified precision (25 ps). Generally the non-linearity specification is a topical task for most high-precision timing devices. However, to specify the non-linearity even generally it is necessary to previously evaluate the function $E(T)$ in details. Moreover, knowing exactly this function it is possible to minimize the non-linearity by its correction in event measurements post-processing.

There are various possible methods of linearity evaluation. The most widespread comparison method suggests that the device under test and some reference device having surely better linearity measure the same time intervals simultaneously [4-5]. In this case the mean of differences between paired measurements of some fixed time interval will characterize specific non-linearity error for this interval. In more detail this method is illustrated in [5] by the example of specific comparison tests at Graz SLR station. Advantage of the comparison method is that it does not need high accuracy of generating test time intervals because wide-range generation of such intervals is not much simpler than their accurate measurement.

However the reference timing devices that provide required high linearity are often inaccessible. Specifically, the timing devices with non-linearity in picosecond range (such as Event Timing System of Graz SLR station [6] that guarantees 2-3 ps non-linearity) currently are unique and their cost is very high. And how to evaluate the linearity for devices with the same or better non-linearity and not only evaluate but get the function $E(T)$ in specified range of time interval variation? Thus the problem to be solved is to find a method which both provides high reliability of linearity evaluation in picosecond range and is simple enough to implement it.

Statistical method of linearity evaluation

Generally there are various reasons for non-linearity. For the event timers the main reason is a recovery process in electrical circuits responsible for event measurement. If the recovery process after the previous measurement is not completed the next measurement may be performed with some non-linearity error. Correspondingly it can be believed that non-linearity error is quasi zero when the time interval between adjacent events surely exceeds recovery time. Also it is assumed that the recovery process is well reproducible independently of prehistory of each event measurement. These assumptions are essential for further consideration.

Let's consider the test circuit (Fig. 1) where the generator A generates a periodic pulse sequence A with a constant period T_A surely greater than the recovery time, and the generator B generates a pulse sequence B with the period 4-5 times greater than that of the generator A.

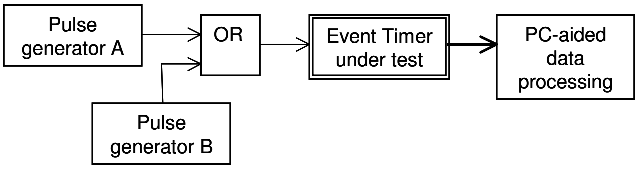

Fig.1. Test circuits for linearity evaluation

As for stability of these sequences, there are no special requirements for that, except for independence of their generation and short-term stability (low jitter) for the sequence A. The most of present-day event timers offer a few identical inputs and mark measured events in accordance with the inputs where they come. This facility simplifies both the test circuit and following data processing.

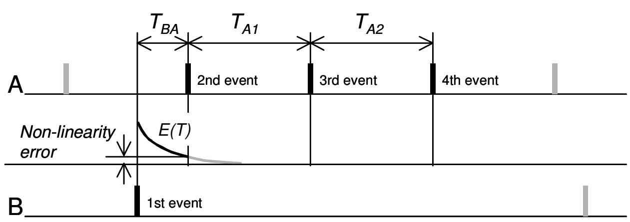

Fig.2. Time diagram of measured events arriving

Under above conditions within sequence of measurement results there are always series of four sequential measurements as shown in Fig.2. In this case the incomplete recovery process can distort the measurement of the second event after the first event measurement when the third and fourth events are measured without non-linearity errors by definition. As a result there is a difference between measured time intervals T_{A1} and T_{A2} which characterizes the non-linearity error for specific value of time interval T_{AB}. More exactly, the difference (T_{A1}-T_{A2}) will conform to the value $E(T_{BA})+\xi$ that contains a search value of non-linearity error in combination with a random error.

Another similar series will reflect the same variables but for some other, naturally randomized values of time interval T_{BA}. When the number of such series is large enough, the values of time interval T_{BA} will have nearly uniform distribution resulting in statistical presentation of the function *E(T)*. Note that such randomized defining of time intervals T_{BA} makes the evaluation process independent of frequency drift for the test pulse sequences.

However such presentation provides randomly distributed estimates of the function *E(T)*, which in addition are considerably corrupted by the random errors. To get the monotone smooth function *E(T)* for further applications it is necessary to average these estimates within some constant increments τ of increasing the time interval T_{BA}. Such procedure will result in the uniformly sampled function $E^*(T)$, where each point reflects a mean of non-linearity error within the step of grid equal to the predetermined increment value τ.

Although the evaluated in this way function $E^*(T)$ is defined in the full region of its existence, typically a great number of initial estimates should be obtained to reduce the evaluation error down to acceptable value. As it usually is, the evaluation error directly depends on the amount of initial samples being averaged. For example, to reduce the evaluation error to one-tenth (as compared to the error of single measurement) at least 200 initial estimates should be averaged within each increment. On the other hand the increment value cannot be too large since the averaging represents a specific filtering and some important details of the actual non-linearity may be lost. Generally a number of initial estimates should be as greater as possible in view of available test duration. For example in our practice the number of initial estimates frequently reaches hundreds of millions. Collecting of such statistics needs a few days of continuous tests.

Summarizing the features of the considered method, let's note its basic advantages and limitation.

Advantages:

- The method is quite simple for implementation and can be easily computerized; no specific expensive equipment is needed; there are no strong requirements to test signal stability.
- The method allows detecting the non-linearity errors in a wide range with precision that depends only on the volume of statistics. Specifically, the method is quite practicable for linearity evaluation with picosecond precision which is quite enough for the most advanced event timers.

Limitations:

- To provide the required volume of statistics, the test duration may be too long.
- The accepted above assumption concerning stability of the recovery process (independently of prehistory for every event measurement) is not always applicable. Generally the non-linearity error of the event measurement may depend on the specific location of a whole series of previous events. In this case the

evaluation result may be obtained with certain distortions.

Example of the method application

We have been applying the considered method for long time in the framework of development of various high-resolution event timers. Currently this method is provided by specialized test software allowing automatic testing of the event timers during practically unlimited time. Specifically, this software provides cyclically:

- Detection of the series measurement results (as Fig. 2 suggests)
- Calculation of a single non-linearity error estimate
- Matching the obtained estimate with specific sub-range of time interval incrementing
- Cumulative averaging of this estimate and similar estimates previously obtained for this sub-range.

The current view of evaluated non-linearity function is periodically displayed so that the process of its creation could be observed in real time. The test software is able to support the evaluation test during the long time, depending only on accessible computer memory.

Typical results of the method application are illustrated below for linearity evaluation of the latest model of Riga Event Timer A032-ET [7]. This instrument provides <10 ps RMS resolution and non-linearity that does not exceed a few picoseconds. However such high linearity to a great extent was achieved by accurate evaluation of initial non-linearity and its further correction.

The basic test was performed for the time interval range to 11.296 μs with 1 ns increment. The total number of single estimates was about 238.187 millions (about 21086 estimates for each increment step). Such statistics have been collected during 64.26 hours of continuous testing. Fig. 3 shows the amount of estimated time intervals T_{BA} for each 1 ns increment. As can be seen, the distribution is nearly uniform, except the range up to 60 ns which is distorted by the “dead time” of event measurement. Note that the test result clearly indicates actual “dead time” value for event timing.

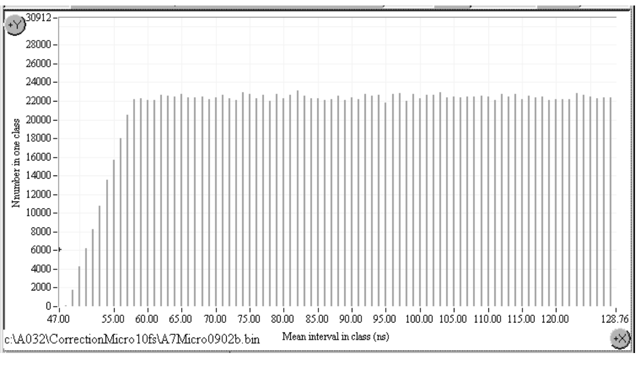

Fig.3. The amount of estimated time intervals for each 1 ns increment at the beginning of the test range

Fig. 4 shows general view of the linearity function evaluated under above test conditions. In view of statistics volume and actual value of random errors of every single estimate the calculated standard deviation of evaluation error is about 0.15 ps.

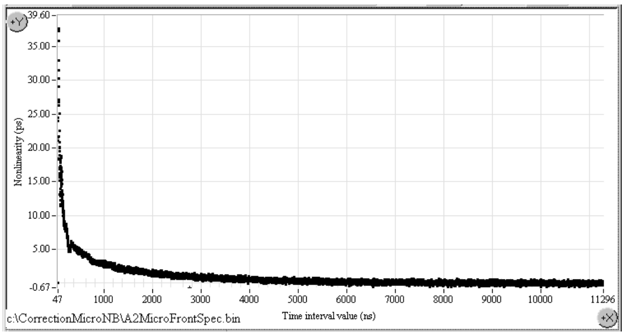

Fig.4. Evaluated linearity function for full range (11.2 μs)

As would be expected, the non-linearity is most essential in the beginning of range (up to 0.5 μs, Fig. 5), then it is decreasing and reaches nearly zero for time intervals greater than 9-10 μs. In general this view fully conforms to the assumption concerning recovery process influence.

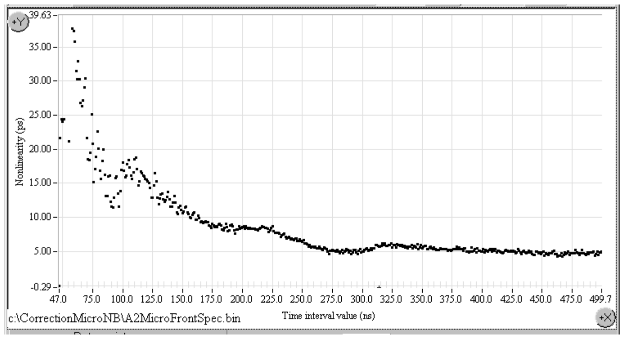

Fig.5. Linearity function for intervals in range up to 0.5 μs

The simplest way of specifying the actual linearity of the device from the evaluation result is to define the maximum non-linearity error in the related sub-range of time interval variation. However, since the Event Timer A032-ET is a virtual (computer-based) instrument, this evaluation result has been directly used for non-linearity correction by the software means.

General correction procedure is based on the specific correction of measurement result for every event, depending on the time interval between this event and the previous one. The correction function is defined in a table format directly on the basis of the evaluation results. However the uncertainty of such function caused by random errors of its evaluation results in some uncorrectable residual non-linearity. To minimize this non-linearity the correction function has been defined for two sub-ranges with two different increments. In the sub-range below 2 μs (where the initial non-linearity is the most essential due to non-damped transients in electrical circuits) the correction function was defined with 1 ns step. In the sub-range above 2 μs (where the non-linearity function becomes monotone and smooth) the correction function was defined with much greater (256 ns) step to decrease the evaluation error. Fig. 6 shows the result of linearity evaluation under similar to the above test conditions but when the described correction is applied.

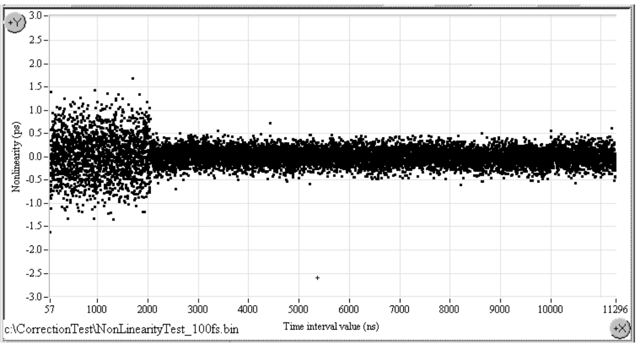

Fig.6. Evaluated linearity function after correction

As can be seen, the residual non-linearity is about 1 ps in the first correction sub-range and less than 0.5 ps in the second sub-range. In the range exceeding the correction range the non-linearity is actually absent.

Conclusions

1. The considered method allows evaluating the actual linearity of high-precision event timers without use of special expensive test equipment. As the independent comparison tests suggest [5], the method provides quite reliable results of linearity evaluation.
2. The linearity functions evaluated in this way can be successfully used for correction of initial non-linearity to reduce it down to picoseconds.

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Systematic errors that depend on the measured values characterize the measurement linearity. Considered method of linearity evaluation for event timers is based on random perturbations of periodic event timing and further measurement of errors caused by such perturbations. The method allows evaluating the linearity with sub-picosecond precision in a wide range of time interval variation, without use of special and expensive equipment. Example of the method application for linearity evaluation of picosecond-precision event timer is considered. Ill. 6, bibl. 7 (in English; summaries in English, Russian and Lithuanian).

Ю. Артюх, В. Беспалько, Е. Буль. Статистический подход к оценке линейности высокоточных таймеров событий // Электроника и электротехника. – Каунас: Технология, 2006. № 4(68). – С. 72–75.

Систематические ошибки, зависящие от значений измеряемых величин, характеризуют линейность измерения. Рассмотрен метод оценки линейности таймеров событий, основанный на случайных возмущениях периодического процесса таймирования и измерении вносимых этими возмущениями ошибок. Метод позволяет оценивать с субпикосекундной точностью линейность в широком диапазоне изменения временных интервалов между событиями без примения для этого специального дорогостоящего оборудования. Приведен пример применения метода для оценки линейности таймера событий пикосекундной точности. Ил. 6, библ. 7 (на английском языке; рефераты на английском, русском и литовском яз.).

J. Artiuch, V. Bespalko, E. Bol. Statistinis aukšto tikslumo įvykių laikmačių tiesiškumo įvertinimas // Elektrotechnika ir elektronika. – Kaunas: Technologija, 2006. – Nr. 4(68). – P. 72–75.

Nuo išmatuotų verčių priklausančios sisteminės paklaidos apibūdina matavimo tiesiškumą. Nagrinėjamas įvykių laikmačių tiesiškumo įvertinimo metodas, kuris remiasi atsitiktiniais laiko paskyrimo periodiniams procesams trikdžiais ir tolimesniu paklaidų, sąlygotų tokiais trikdžiais, matavimu. Metodas leidžia įvertinti teisiškumą didesniu nei pikosekundės tikslumu plačiame laiko intervalo kitimo diapazone nenaudojant specializuotos ir brangios įrangos. Pateiktas metodo taikymo pikosekundės tikslumo laikmačio tiesiškumui įvertinti pavyzdys. Il. 6, bibl. 7 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).